Modeling, Analysis, and Optimization of Cascaded Power Amplifiers

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Abstract—This paper deals with modeling, analysis, and optimization of power amplifiers (PAs) placed in a cascaded structure, particularly the effect of cascaded nonlinearities is studied by showing potential ways to minimize the total nonlinearities. The nonlinear least-squares algorithm is proposed to optimize the PA parameters along with the input power level, and thereby minimize the total nonlinearities in the cascaded structure. The simulation results demonstrate that the performance of the optimized configurations for up to five PAs using the proposed framework can improve the linearity properties of the overall cascade.

Index Terms—Power amplifier nonlinearity, cascaded structure, minimization of total nonlinearities, nonlinear least-squares

I. INTRODUCTION

Radio frequency power amplifiers (PAs) are essential elements in wireless communication systems, but they are also one of the main sources of nonlinearities that distort the transmitted signal and degrade the overall system performance. This is especially severe in systems operating on signals with high peak-to-average power ratio (PAPR). In order to design high-performance wireless transmitters, the behavioral modelling of PAs has received considerable attention over the last few decades, as it is an essential step in understanding PAs limitations and developing efficient digital signal processing algorithms for optimizing their performance [1]–[4].

Recent advances in wireless communication systems have shown the need for considering a cascade of PAs, where each of them amplifies the incoming signal to compensate for the losses incurred in the preceding fiber [5]. A similar problem has been considered in optical free-space communications [6] and optical communications [7] to overcome the impact of the attenuation for long distances. Additionally, cascaded structures of PAs have been proposed to improve power efficiency and minimize chip size [8], [9]. However, the effect of cascaded nonlinearities has not been fully studied in these areas, moreover, many papers assumed that the nonlinear effect is negligible.

Further, requirements on implementation complexity and power consumption are growing with the evolution of communication systems. It is usually desirable that a PA operates near saturation region in order to achieve maximum power efficiency, which is especially important in ultra-highfrequency communication systems. There has been a large amount of literature on modeling a single PA and various techniques on linearization, including digital pre-distortion at the transmitter [10], [11] and digital post-distortion at the receiver [12], proposed to achieve a balance between linearity, adequate amplification, and implementation complexity. However, in the case of cascaded PAs, the distortions from each amplifier accumulate with those from the preceding stages, leading to severe nonlinear behavior. Moreover, since the PA nonlinearity strongly depends on the input power, the behavior of each PA in the cascade is influenced by the output of the preceding PA. This highlights an important research direction for investigating the behavior of cascaded PAs and discovering potential ways to minimize the effect of the cascaded structure.

In this paper, we address this issue by modeling the cascaded PAs, providing a thorough analysis of total nonlinearities, and optimizing the parameters of the cascade to mitigate the severe effect of the cascaded amplifiers. The main contributions of the paper are as follows.

- An analysis of the nonlinearities in the cascaded PAs structure is presented, showing potential ways of minimizing the effect of cascading.
- A nonlinear least-squares (NLS) optimization problem is formulated and solved to minimize the total nonlinearities in the cascaded structure by optimizing the PA gain values and input power.
- Numerical results demonstrating the performance of the optimized configurations using the proposed framework are presented for up to five PAs in cascade.

Following the introduction, Section II introduces the model of the cascaded PAs. In Section III, an analysis of nonlinearities in the cascaded structure is given and the ways of minimizing them are highlighted. The optimization problem to minimize the total nonlinearities is presented in Section IV, following simulation results provided in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

Due to inherent nonlinear behavior of PAs in practice, they introduce nonlinear distortion to the incoming signal with the increase of power. To analyse the effect of total



Fig. 1. Model of the cascaded PAs.

nonlinearities in the cascaded structure, where the distortions are accumulated with the increase in the number of amplifiers, the cascaded PAs can be modeled as a cascade of nonlinear functions indicating the PA behavior and noise sources representing thermal noise, as it is shown in Fig. 1.

We assume that the complex baseband equivalent of the input signal to the first PA in the cascade is denoted by $x_n^{(0)}$, with power of p_0 . PA behavior under normal operation conditions, in the simplest case, can be modeled using the third-order memoryless polynomial model [3]. Thus, the output signal $y_n^{(k)}$ of the *k*th PA can be described as

$$y_n^{(k)} = g^{(k)} f(x_n^{(k)}) = g^{(k)} (x_n^{(k)} + \alpha^{(k)} x_n^{(k)} |x_n^{(k)}|^2),$$
(1)

where $x_n^{(k)}$ is the input signal to the kth PA, $\alpha^{(k)}$ is the thirdorder nonlinearity coefficient, which is typically complexvalued to capture both amplitude-to-amplitude modulation (AM/AM) and amplitude-to-phase modulation (AM/PM). Since third-order nonlinearities are generally dominant compared to higher-order terms in the context of a single PA [3], the model in (1) captures the majority of nonlinear distortions and can be used for further analysis of a cascade of PAs. Additionally, in this paper, we assume that the amplifiers are placed in cascade to compensate for the losses incurred in the connectors between them. Therefore, the coefficient $g^{(k)}$ represents both linear amplification of the PA and attenuation due to losses in the connector. In the case when linear amplification exactly equals attenuation, $q^{(k)} = 1$. Furthermore, the thermal noise between the PAs is modeled as additive white Gaussian noise (AWGN) with the variance σ^2 .

To express the output of the (k + 1)th PA, a function incorporating a single stage, represented by the PA nonlinearity and thermal noise, is defined as

$$F(y_n^{(k)}, g^{(k+1)}) = g^{(k+1)} f(y_n^{(k)} + \sigma w_n^{(k+1)}).$$
(2)

Then, for the signal $x_n^{(0)}$ passing through K cascaded PAs, the output $y_n^{(K)}$ can be expressed by applying the function $F(\cdot)$ in (2) iteratively K times, i.e.,

$$y_n^{(K)} = \underbrace{F(F(\dots F(x_n^{(0)}, g^{(1)}), g^{(2)})\dots)}_{K \text{ times}}.$$
 (3)

III. ANALYSIS OF NONLINEARITIES IN THE CASCADED STRUCTURE

In this section, we focus on the analysis of nonlinearities accumulating in the cascaded structure to show potential ways for minimizing the total distortions by optimizing parameters of the cascaded structure, as will be presented in Section IV.

A. Cascaded Nonlinearities in the Noise-Free Case

In the noise-free case ($\sigma = 0$), the output of the second PA can be written in terms of $x_n^{(0)}$ using (3) as

$$y_n^{(2)} = g^{(2)} \left[g^{(1)} \left(x_n^{(0)} + \alpha^{(1)} x_n^{(0)} |x_n^{(0)}|^2 \right) + \alpha^{(2)} \left(g^{(1)} \right)^3 \left(x_n^{(0)} + \alpha^{(1)} x_n^{(0)} |x_n^{(0)}|^2 \right) \\ \times \left| x_n^{(0)} + \alpha^{(1)} x_n^{(0)} |x_n^{(0)}|^2 \right|^2 \right].$$
(4)

Assuming that $\alpha^{(k)}$, k = 1, ..., K, are small values, we can approximate the quadratic terms $\alpha^{(k)}\alpha^{(l)}$ for k, l = 1, 2, ..., K as being negligibly small. Consequently, the terms in (4) containing these coefficients can be omitted from this analysis. Then, $y_n^{(2)}$ in (4) can be approximated by $\tilde{y}_n^{(2)}$ given by

$$\tilde{y}_{n}^{(2)} = g^{(2)}g^{(1)} \bigg[x_{n}^{(0)} + \bigg(\alpha^{(1)} + \alpha^{(2)} \big(g^{(1)} \big)^{2} \bigg) x_{n}^{(0)} |x_{n}^{(0)}|^{2} \bigg].$$
(5)

Applying the same calculations and utilizing the assumption $\alpha^{(k)}\alpha^{(l)} \approx 0$ for $k, l = 1, 2, \ldots, K$, the output of the third PA can be expressed as

$$\tilde{y}_{n}^{(3)} = g^{(3)}g^{(2)}g^{(1)} \left[x_{n}^{(0)} + \left(\alpha^{(1)} + \alpha^{(2)} \left(g^{(1)} \right)^{2} + \alpha^{(3)} \left(g^{(2)}g^{(1)} \right)^{2} \right) x_{n}^{(0)} |x_{n}^{(0)}|^{2} \right].$$
(6)

Following the same procedure, the Kth PA output can be approximated by

$$\tilde{y}_{n}^{(K)} = \tilde{g}(\mathbf{g}, K) \Big[x_{n}^{(0)} + \tilde{\alpha}(\mathbf{g}, \boldsymbol{\alpha}, K) x_{n}^{(0)} |x_{n}^{(0)}|^{2} \Big],$$
(7)

where $\tilde{g}(\mathbf{g}, K)$ and $\tilde{\alpha}(\mathbf{g}, \boldsymbol{\alpha}, K)$ can be seen as the gain and third-order nonlinearity coefficient of an equivalent PA, given respectively by

$$\tilde{g}(\mathbf{g}, K) = \prod_{k=1}^{K} g^{(k)},\tag{8}$$

$$\tilde{\alpha}(\mathbf{g}, \boldsymbol{\alpha}, K) = \alpha^{(1)} + \sum_{k=2}^{K} \alpha^{(k)} \prod_{q=1}^{k-1} (g^{(q)})^2, \qquad (9)$$

with $\mathbf{g} = [g^{(1)}, g^{(2)}, \dots, g^{(K)}]$ and $\boldsymbol{\alpha} = [\alpha^{(1)}, \alpha^{(2)}, \dots, \alpha^{(K)}].$

B. Cascaded Nonlinearities in the Presence of Noise

In the presence of noise with variance σ^2 , the input to the kth PA, $k = 1, 2, \ldots, K$, contains the additional component $\sigma w_n^{(k)}$. Then, following the same analysis as in Section III-A and assuming that σ is also a small value, the quadratic terms $\alpha^{(k)}\alpha^{(l)}$, $\alpha^{(k)}\sigma$, σ^2 , for $k, l = 1, 2, \ldots, K$, can be approximated as being negligibly small. Then, the output of the Kth PA can be approximated by

$$\tilde{y}_{n}^{(K)} = \tilde{g}(\mathbf{g}, K) \left[x_{n}^{(0)} + \tilde{\alpha}(\mathbf{g}, \boldsymbol{\alpha}, K) x_{n}^{(0)} |x_{n}^{(0)}|^{2} \right] + \tilde{\sigma}(\mathbf{g}, K) w_{n},$$
(10)

where $\tilde{\sigma}(\mathbf{g}, \mathbf{a}, K)$ is the standard deviation of an equivalent AWGN noise after the *K*th PA given by

$$\tilde{\sigma}(\mathbf{g}, K) = \sigma \sqrt{\sum_{k=1}^{K} \prod_{q=k}^{K} (g^{(q)})^2}.$$
(11)

C. Analysis of the Approximating Expressions

From equations (7)–(11), it can be observed that the amount of distortions at the output of the cascade of K PAs strongly depend on their parameters $g^{(k)}$ and $\alpha^{(k)}$, $k = 1, \ldots, K$, and the values of the input signal $x_n^{(0)}$ to the first stage, i.e., the power p_0 . Specifically, a linear combination of $\alpha^{(k)}$ in (9) relies on the product of quadratic terms $(q^{(k)})^2$, and noise level in (11) is influenced by the product of the linear terms $g^{(k)}$. The coefficient $\alpha^{(k)}$ represents the inherent nonlinear distortion characteristics of the kth amplifier, that is generally difficult to change without redesigning the amplifier. In contrast, the input power p_0 and linear gain $q^{(k)}$ can be more easily adjusted in practice. In this paper, we consider that the PAs in cascade have the same nonlinear characteristics, i.e., $\alpha^{(k)} = \alpha$. Thus, varying the parameters p_0 and $g^{(k)}$ can lead to minimizing the total nonlinearities at the output of the Kth PA.

One of the challenges lies in finding a compromise between maintaining the desired linear gain at the output of the *K*th PA and minimizing the overall distortions because reducing the gain $g^{(k)}$ in (9) can also decrease the total linear amplification term in (8) and can affect the noise component in (11).

It is important to emphasize that equations (7)–(11) were derived to provide insights into the total nonlinearities and to highlight the need for optimizing the parameters of the cascaded structure in order to minimize the overall distortions. Further, the cascaded function in (3) will be utilized in the optimization problem presented in the next section.

IV. OPTIMIZATION PROBLEM

A. Problem Formulation

To tackle the problem of minimizing the total nonlinearities in the cascaded PAs structure, an NLS algorithm is employed in this paper. The problem is to minimize the total nonlinearities by optimizing the PA gains $g^{(k)}$ and input power p_0 , i.e.,

$$\|Gx_n^{(0)}/\sqrt{p_0} - y_n^{(K)}\|^2 \tag{12}$$

subject to $0 < p_0 \le 1$,

 $\min_{p_0,\mathbf{g}}$

$$(1-\varepsilon)G \le g^{(k)} \le (1+\varepsilon)G, k = 1, 2, \dots, K,$$
(13)

where G is the reference gain (desired) at the output of the Kth PA, $y_n^{(K)}$ as in (3), and ε is the parameter determining the allowable range for the PA gains $g^{(k)}$. Here, we consider two following sub-cases, that add additional constraints to (13):

- (a) Equal PA gains, i.e., $g^{(k)} = g$ and $\mathbf{g} = [g, g, \dots, g]$. Then, the optimization parameters are p_0 and g in (12).
- (b) Unequal PA gains, i.e., the optimization parameters are p_0 and $\mathbf{g} = [g^{(1)}, g^{(2)}, \dots, g^{(K)}]$ in (12).

Note that the normalization by $1/\sqrt{p_0}$ in (12) eliminates the dependence of the desired signal on the input power p_0 .

B. Performance Metrics

The performance of the cascaded PA models with optimized parameters is evaluated in terms of normalized mean square error (NMSE) and adjacent channel leakage ratio (ACLR). The NMSE is defined as

NMSE =
$$\frac{\sum_{n=0}^{N} |y_n - y_n^{(K)}|^2}{\sum_{n=0}^{N} |y_n|^2}$$
, (14)

where $y_n = Gx_n^{(0)}/\sqrt{p_0}$ is the desired output and $y_n^{(K)}$ is the output of the Kth PA in (3).

The ACLR is defined as

ACLR =
$$\frac{\int_{\text{max. adj.}} |Y^{(K)}(f)|^2}{\int_{\text{ch.}} |Y^{(K)}(f)|^2}$$
, (15)

where $Y^{(K)}(f)$ is the power spectrum of the signal $y_n^{(K)}$. Here, the integration in the nominator is performed over the adjacent channel with the highest power, while the denominator's integration is carried out over the main channel.

V. SIMULATION RESULTS

A. The Choice of the Starting Point for the Optimization

Since the optimization problem in (12) is nonlinear, a reasonable starting point is needed to prevent the convergence to a poor local optimum. Thus, we consider two initial scenarios:

- (1) The linear gain of every PA corresponds to the losses that occurred in the connector, i.e., $g^{(k)} = 1, k = 1, 2, \dots, K$.
- (2) Every PA operates in the most efficient regime, i.e., with the maximum input power, and in this case, $g^{(k)} = g = x_{\text{max}}/f(x_{\text{max}})$ representing amplification to achieve the maximum input level of one, with $x_{\text{max}} = \sqrt{1/(3|\alpha|)}$.

A cascade of $K = 1, \ldots, 5$ PA(s) is considered for both scenarios, with $\alpha = -0.33(1-0.1j)$, $p_0 = 1$ and $\sigma^2 = 10^{-5}$. The input signal $x_n^{(0)}$ is a normalized 16QAM complex baseband signal. The desired linear gain G is set to one. The



Fig. 2. AM/AM of the cascaded PAs for the two initial scenarios.



Fig. 3. PSD of the cascaded PAs for the two initial scenarios.

AM/AM characteristics and the power spectral density (PSD) of the Kth PA output signals for the two initial scenarios are shown in Figs. 2 and 3, respectively. Clearly, as K increases, the level of distortion rises. In Scenario 1, the output signal of each successive PA accumulates nonlinear distortions and experiences saturation, resulting in a progressive loss of the initial amplitude as the number of cascaded PAs increases. In Scenario 2, the output signal becomes increasingly susceptible to nonlinear distortions due to enhanced nonlinear effects, while its amplitude remains approximately unchanged. The results in terms of NMSE and ACLR are shown in Fig. 4, where it seen that the first scenario shows better performance, and thus can serve as the initial starting point for further optimization. Moreover, these two scenarios can serve as the upper and lower bounds for choosing a starting point to solve the nonlinear optimization problem in (12).

B. Optimization of the Parameters in the Cascaded Structures

1) Input Power Optimization: First, to investigate how the input power p_0 influences the performance, the problem in (12) is considered by optimizing a single coefficient p_0 , given fixed PA gain coefficients **g**. The values of the optimized input signal power are listed in Table I, and the performance of the optimized configurations are shown in Fig. 4. It is seen

TABLE I INPUT SIGNAL POWER AFTER OPTIMIZATION.

	K PAs in a cascade						
Case	1	2	3	4	5		
Input power opt. (Scenario 1)	1	1	1	1	1		
Input power opt. (Scenario 2)	0.49	0.23	0.11	0.05	0.02		
Input power and equal PA gains optimization	0.65	0.40	0.25	0.19	0.19		
Input power and unequal PA gains optimization	0.65	0.40	0.27	0.30	0.32		

 TABLE II

 PA GAIN VALUES AFTER OPTIMIZATION.

		K PAs in a cascade						
Case	k	1	2	3	4	5		
Equal PA gains optimization	1	1.07	1.07	1.07	1.06	1.06		
	2	-	1.07	1.07	1.06	1.06		
	3	-	-	1.07	1.06	1.06		
	4	-	-	-	1.06	1.06		
	5	-	-	-	-	1.06		
Unequal PA gains optimization	1	1.07	0.86	0.70	0.70	0.70		
	2	-	1.30	1.26	0.99	0.82		
	3	-	-	1.30	1.30	1.21		
	4	-	-	-	1.30	1.30		
	5	-	-	-	-	1.30		
Input power and equal PA gains optimization	1	1.30	1.30	1.30	1.26	1.21		
	2	-	1.30	1.30	1.26	1.21		
	3	-	-	1.30	1.26	1.21		
	4	-	-	-	1.26	1.21		
	5	-	-	-	-	1.21		
Input power and unequal PA gains optimization	1	1.30	1.30	1.24	1.02	1.02		
	2	-	1.30	1.30	1.17	1.03		
	3	-	-	1.30	1.30	1.13		
	4	-	-	-	1.30	1.29		
	5	-	-	-	-	1.30		

that $p_0 = 1$ is the optimal value for Scenario 1, while it is not for Scenario 2 where the performance has improved after optimization, and even outperforms Scenario 1. However, as the number of PAs in the cascade increases, the input power in Scenario 2 decreases significantly, potentially leading to a reduced SNR in the initial stages due to the constant thermal noise power.

2) PA Gains Optimization: Further, to investigate how the PA gains can be optimized given maximum input power $p_0 = 1$, the problem in (12) is considered with equal gains, i.e., by optimizing a single coefficient g, and with unequal gains, i.e., by optimizing coefficients $g^{(1)}, g^{(2)}, \ldots, g^{(K)}$. Scenario 1 is chosen as the initial starting point, and the parameter $\varepsilon = 0.3$ is set here. The results shown in Fig. 4 indicate that the case with unequal gains exhibits better performance. Notably, the difference between the two cases becomes more pronounced as the number of PAs in the cascade increases. The values of the optimized PA gain values are listed in Table II.

3) Joint Input Power and PA Gains Optimization: Finally, joint optimization of the PA gains and input power is considered here. Scenario 1 as the starting point and $\varepsilon = 0.3$ are also chosen here. The results of the optimization are shown in Figs. 4–6, and the optimized values of the input signal power and PA gains are listed in Tables I and II,



Fig. 4. Results of the optimization of the cascaded PAs parameters in terms of NMSE and ACLR.



Fig. 5. AM/AM of the output after K PAs with the optimized input power and PA gains (equal and unequal gains).



Fig. 6. PSD of the output after K PAs with the optimized input power and PA gains (equal and unequal gains).

respectively. One can observe that incorporating the input power level into the optimization, along with the gains, further enhances the potential for reducing overall distortions and mitigating the severe effects of cascading K PAs, however, the difference between equal and unequal gains is small and becomes noticeable only for $K \ge 4$.

VI. CONCLUSIONS

This paper studied modeling of cascaded PAs and provided an analysis of the total nonlinearities, which accumulate as the number of PAs increases. The NLS algorithm was proposed to optimize PA gains and input power levels, thereby minimizing the overall impact of cascaded nonlinearities. Simulation results demonstrated the performance improvement of cascaded structures optimized using the proposed framework.

The analysis presented in this paper can provide opportunities to develop efficient linearization techniques while achieving an optimal balance between minimizing overall distortions, enhancing system performance, and reducing implementation complexity. In future work, higher-order polynomials can be utilized to model cascaded PAs more accurately, and digital predistortion techniques can be investigated to enhance the performance of cascaded PA structures.

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